Entanglement-preserving frequency conversion in cold atoms

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We propose a method that enables efficient frequency conversion of quantum information based on recently demonstrated strong parametric coupling between two single-photon pulses propagating in a slow-light atomic medium at different group velocities. We show that an incoming single-photon state is efficiently converted into another optical mode in a lossless and shape-conserving manner. The persistence of initial quantum coherence and entanglement within frequency conversion is also demonstrated. We first illustrate this result for the case of small frequency difference of converted photons, and then discuss the modified scheme for conversion of photon wavelengths in different spectral ranges. Finally we analyze the generation of a narrow-band single-photon frequency-entangled state.

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I. INTRODUCTION

Frequency conversion of quantum states of light and redistribution of optical information between different quantum fields is an important and desirable tool for interfacing of quantum communication lines with photon memory units. Quantum frequency conversion (QFC) with preserving a quantum state has been hitherto realized in the nonlinear crystals using the process of parametric up-conversion [1, 2, 3, 4, 5]. For implementation of QFC in atomic ensembles, a technique for light storage and its subsequent retrieval [6] at another optical frequency under the conditions of electromagnetically induced transparency (EIT) [7] was employed. However, despite the success of proof-of-principle experiments performed in a four-level double Λ-type atomic medium [8, 9], to date no true demonstration of information-preserving frequency conversion has been given. The major difficulties inherent to these schemes are unavoidable losses and shape distortion of a weak (quantum) light pulse during its storage and retrieval by means of EIT.

In this paper, we propose a protocol for QFC in atomic ensembles free from the above drawbacks. Our method makes use of recently demonstrated [10] efficient parametric coupling between two single-photon pulses propagating in a slow-light medium. The latter is an ensemble of atoms which interact with two quantum fields in a V-type configuration, while the upper electric-dipole forbidden transition is driven by a classical and constant field inducing a magnetic dipole or an electric quadrupole transition between the two upper levels (Fig.1). The role of classical field is twofold. First, it creates parametric coupling between the photons and, second, it induces medium transparency for the intensity values where the EIT conditions are fulfilled for both quantum fields. It is clear, however, that implementation of the medium transparency leads evidently to degradation of parametric interaction between the fields. Nevertheless, we demonstrate below how an incoming signal-photon state is efficiently converted under EIT conditions into second optical mode in a lossless manner conserving at the same time the pulse shape and initial entanglement. The easiest way to realize this is obviously the case of equal group velocities of quantum fields, which, however, is almost never met in practice (see below). Meanwhile, for realistic atomic systems, we predict the conversion efficiency as high as $\sim 90\%$. In essence, while the process involves only one photon at a time, this system allows for the realization of strong interaction of individual photons with each other, even if the two pulses propagate in the medium with different group velocities. Our approach offers another important advantage: unlike the previous proposals for QFC based on the light storage and its retrieval, we apply here only one and, moreover, constant driving field for control the conversion efficiency that makes our method much favorable for future applications. One obvious limitation of considered process is that it is effective in a relatively narrow frequency range associated with specific atoms. We note, however, that recently a source of narrow-band, frequency tunable single photons with properties allowing exciting the narrow atomic resonances has been created [11, 12, 13, 14].

In next section we briefly describe a three-level model for parametric interaction between two quantum fields

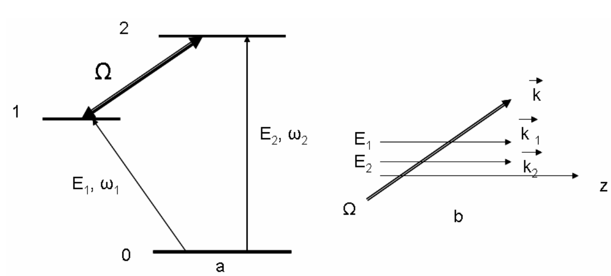

FIG. 1: (a) Level scheme of atoms interacting with quantum fields $E_{1,2}$ and classical driving field of Rabi frequency Ω. (b) Geometry of fields propagation.

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and give the analytic solution for the field operators obtained in [10]. Then, in section III we calculate the intensities of quantum fields and find the conversion efficiency, as well as discuss the ability of the system to convert coherently the photons with wavelengths in different spectral ranges. In section IV we analyze the quantum properties of frequency conversion process and show how a partial conversion leads to generation of frequency entangled single-photon state. Finally, in section IV we summarize our results.

II. PARAMETRIC INTERACTION BETWEEN TWO SINGLE-PHOTON PULSES

We outline here an approach that allows the propagation dynamics to be solved exactly, while the detailed description of the model can be found in our previous paper [10]. We consider an ensemble of cold atoms with level configuration depicted in Fig.1. Two quantum fields

$$E_{1,2}(z,t)=\sqrt{\frac{\hbar\omega_{1,2}}{2\varepsilon_0 V}}\hat{\mathcal{E}}_{1,2}(z,t)\exp[i(k_{1,2}z-\omega_{1,2}t)]+h.c.$$

co-propagate along the z axis and interact with the atoms on the transitions $0\rightarrow 1$ and $0\rightarrow 2$, respectively, while the upper electric-dipole forbidden transition $1\rightarrow 2$ is driven by a classical and constant field with real Rabi frequency Ω, and V is the quantization volume taken to be equal to interaction volume. The electric fields are expressed in terms of the operators $\hat{\mathcal{E}}_i(z,t)$, and the medium is described using atomic operators $\hat{\sigma}_{\alpha\beta}(z,t)=\frac{1}{N_z}\sum_{i=1}^{N_z}\mid\alpha\rangle_i\langle\beta\mid$ averaged over the volume containing many atoms $N_z=\frac{N}{L}dz\gg 1$ around position z, where N is the total number of atoms and L is the length of the medium. In the rotating wave picture the interaction Hamiltonian is given by

$$H=-\hbar\frac{N}{L}\int_0^L dz[g_1\hat{\mathcal{E}}_1\hat{\sigma}_{10}e^{ik_1z}+g_2\hat{\mathcal{E}}_2\hat{\sigma}_{20}e^{ik_2z}+\Omega\hat{\sigma}_{21}e^{ik_{\parallel}z} + h.c.] \tag{1}$$

Here $k_{\parallel}=\vec{k}_d\hat{e}_z$ is the projection of the wave-vector of the driving field on the z axis, $g_\alpha=\mu_{\alpha0}\sqrt{\omega_i/(2\hbar\varepsilon_0 V)}$ is the atom-field coupling constants with $\mu_{\alpha\beta}$ being the dipole matrix element of the atomic transition $\alpha\rightarrow\beta$. We assume that the process is running at low temperature in order to avoid the Doppler broadening, which in a cold atomic sample is smaller than all relaxation rates, and consider the case of exactly resonant interaction with all fields.

To implement our QFC scheme in a dense atomic medium, several conditions must be satisfied. First, the photon absorption must be strongly reduced

$$\kappa_i L\ll 1 \tag{2}$$

where $\kappa_i=g_i^2\Gamma_j N/c\Omega^2, i,j=1,2$ and $i\neq j$, are the field absorption coefficients, and $\Gamma_{1,2}$ are the optical transverse relaxation rates involving, apart from natural decay rates $\gamma_{1,2}$ of the excited states 1 and 2, the dephasing rates in corresponding transitions. The latter are caused by atomic collisions and escape of atoms from the laser beam. However, in the ensemble of cold atoms the both effects are negligibly small compared to $\gamma_{1,2}$, so that $\Gamma_{1,2}$=$\gamma_{1,2}/2$. We emphasize that γ_i is a sum of the partial decay rate of i-th upper level to the ground state 0 and the rate of population leak from the i-th level towards the states outside of the system. The limit of Eq.(2) is readily achieved, if the condition of electromagnetically induced transparency (EIT, ref. [7]) $\Omega\gg\Gamma_{1,2}$ is satisfied for both transitions coupled to the weak-fields. Note that the three-level configurations 0-2-1 and 0-1-2 form the Λ- and ladder EIT-systems with the decoherence time Γ_1^{-1} and Γ_2^{-1}, respectively. Second, the initial spectrum of quantum fields should be contained within the EIT window $\Delta\omega_{EIT}=\Omega^2/(\Gamma\sqrt{\alpha})$ [6], resulting in little pulse distortion from absorption, i.e.

$$\Delta\omega_{EIT}T\geq 1 \tag{3}$$

where T is the initial pulse width, $\alpha=\mathcal{N}\sigma L$ is the optical depth, $\sigma=\frac{3}{4\pi}\lambda^2$ is the resonant absorption cross-section, and $\mathcal{N}$ is the atomic number density. It is also desirable that the pulse broadening should be minimal. The source of the latter is the various orders of dispersion, beginning from the group-velocity dispersion proportional to the second time derivative of the fields $\frac{\partial^2}{\partial t^2}\hat{\mathcal{E}}_i(z,t)$. The change in pulse width due to this term after propagating through the medium can be roughly estimated by treating the weak fields for a time classically and ignoring the parametric contribution to broadening. Then, for an initial Gaussian pulse $\mathcal{E}_1(0,t)=\mathcal{E}_0exp(-\frac{2t^2}{T^2})$, the simple calculations yield

$$T_i(L)=T\sqrt{1+\frac{16L}{v_iT^2\Omega}}$$

with $v_i=c\Omega^2/g_i^2N\ll c$ being the pulse group velocity. It is seen that the spreading of the quantum pulses caused by the group-velocity dispersion can be neglected, if

$$\frac{16\tau_i}{T^2\Omega}\leq 1 \tag{4}$$

where $\tau_i=L/v_i$ is the pulse propagation time and, in fact, the pulse delay relative to the pulse which travels the same distance in vacuum. It is worth noting that upon satisfying the condition (4), the pulse broadening due to next orders of dispersion is more suppressed.

In [10] we have shown that under the conditions (2-4) the propagation equations for the quantum field operators, in the slowly varying envelope approximation, take the simple form :

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_1}\frac{\partial}{\partial t}\right)\hat{\mathcal{E}}_1(z,t)=-i\beta\hat{\mathcal{E}}_2 \tag{5}$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_2}\frac{\partial}{\partial t}\right)\hat{\mathcal{E}}_2(z,t)=-i\beta\hat{\mathcal{E}}_1 \qquad (6)$$

where $\beta = g_1 g_2 N/c\Omega$ is the coupling constant of parametric interaction between the quantum fields and decreases with increase of coupling field intensity.

The formal solution of Eqs.(6,7) in the region $0 \leqslant z \leqslant L$ is given by

$$\hat{\mathcal{E}}_i(z,t)=\hat{\mathcal{E}}_i(0,\ t-z/v_i)+\int_0^z dx\{\hat{\mathcal{E}}_i(0,\ t-z/v_j-\frac{\Delta v_{ji}}{v_i v_j}x)$$

$$\times\,\frac{\partial J_0(\psi)}{\partial z}-i\beta\ \hat{\mathcal{E}}_j(0,\ t-z/v_i-\frac{\Delta v_{ij}}{v_i v_j}x)\ J_0(\psi)\} \qquad (7)$$

where $i,j=1,2$ and $j\neq i$. The Bessel function $J_0(\psi)$ depends on z via $\psi=2\beta\sqrt{x(z-x)}$, $\Delta v_{ij}=v_i-v_j$ is the difference of group velocities. In deriving this solution we have assumed that the phase-matching condition $\Delta k = k_2-k_1-k_\parallel=0$ is fulfilled in the medium.

In the limit of equal group velocities $v_2=v_1=v$ the Eqs. (7) are reduced to a simple form

$$\hat{\mathcal{E}}_i(z,t)=\hat{\mathcal{E}}_i(0,\tau)cos(\beta z)-i\hat{\mathcal{E}}_j(0,\tau)sin(\beta z) \qquad (8)$$

where $\tau = t-z/v$, $j\neq i$. Note, however, that this case is practically never realized, since the atom-field coupling constants g_1 and g_2 are defined by the partial decay rates of upper atomic 1 and 2 levels into the ground state 0, which are different even if the total decay rates are the same $\gamma_1=\gamma_2$, as is the case, for example, in a V-type atoms with hyperfine-level structure.

III. QUANTUM EFFICIENCY OF CONVERSION

Our aim is to show that the proposed scheme is suitable for converting individual photons at one frequency to another frequency while preserving initial quantum coherence that results in a generation of frequency-entangled single-photon state. To this end we analyze the evolution of the input state $\mid\psi_{in}\rangle=\mid 1_1\rangle\otimes\mid 0_2\rangle$ consisting of a single-photon wave packet at ω_1 frequency, while ω_2 field is in the vacuum state. The similar results are clearly obtained in the case of one input photon at ω_2 frequency. We assume that initially the ω_1 pulse is localized around $z=0$ with a given temporal profile $f_1(t)$:

$$\langle 0\mid\hat{\mathcal{E}}_1(0,t)\mid\psi_{in}\rangle=\langle 0\mid\hat{\mathcal{E}}_1(0,t)\mid 1_1\rangle=f_1(t) \qquad (9)$$

In free space, $\hat{\mathcal{E}}_1(z,t)=\hat{\mathcal{E}}_1(0,t-z/c)$ and we have

$$\langle 0\mid\hat{\mathcal{E}}_1(0,t-z/c)\mid 1_1\rangle=f_1(t-z/c). \qquad (10)$$

The intensities of the fields at any distance in the region $0\leqslant z\leqslant L$ are given by

$$\langle I_i(z,t)\rangle=\mid\langle 0\mid\hat{\mathcal{E}}_i(z,t)\mid\psi_{in}\rangle\mid^2 \qquad (11)$$

The quantum efficiency (QE) is determined as the ratio $n_2(L)/n_1(0)$, where $n_i(z)=\langle\psi_{in}\mid\hat{n}_i(z)\mid\psi_{in}\rangle$ are the mean photon numbers and $\hat{n}_i(z)$ are the dimensionless operators for number of photons that pass each point on the z axis in the whole time

$$\hat{n}_i(z)=\frac{c}{L}\int dt\hat{\mathcal{E}}_i^+(z,t)\hat{\mathcal{E}}_i(z,t) \qquad (12)$$

Using Eqs.(7, 9-12) and recalling that $\langle 0\mid\hat{\mathcal{E}}_2(0,t)\mid\psi_{in}\rangle=0$, we calculate n_i numerically and show in Fig.2 the QE as a function of driving field Rabi frequency Ω in the case of a Gaussian input pulse $f(t)=C\exp[-2t^2/T^2]$, which is normalized as $\frac{c}{L}\int\mid f_1(t)\mid^2 dt=1$ indicating that the number of impinged photons is one. The atomic sample is chosen to be ^{87}Rb vapor with the ground state $5S_{1/2}(F_g=2)$ and exited states $5P_{1/2}(F_e=1)$, $5P_{3/2}(F_e=1)$ as the atomic states 0 and $1,2$ in Fig.1, respectively. For calculations we used the following parameters: light wavelength of quantum fields λ_1 =795 nm and λ_2 =780 nm, $\Gamma=\Gamma_2=2\Gamma_1=2\pi\times 3$ MHz, atomic density $\mathcal{N}\sim 10^{13}\mathrm{cm}^{-3}$ in a trap of length $L\sim 100\ \mu$m, and the input pulse duration $T\simeq 20$ ns, for which we have $v_1\sim 1.25\cdot 10^4 m/s$, $v_2\sim 0.5v_1$, and $\kappa_i L<0.1$. All of these parameters appear to be within experimental reach, including the initial single-photon wave packets with a pulse length of several tens of nanoseconds [13, 14], as well as high power narrow-band cw THz radiation at a frequency ~ 7 THz [15] resonant to the fine splitting $\omega(5P_{3/2}-5P_{1/2})$ in rubidium.

It is worth noting that in most cases of alkali atoms the spectral separation between $D2$ and $D1$ lines lies in the operation range of tunable terahertz silicon and quantum cascade lasers [16, 17] that may give novel important applications of THz radiation such as implementation of QFC in atomic ensembles.

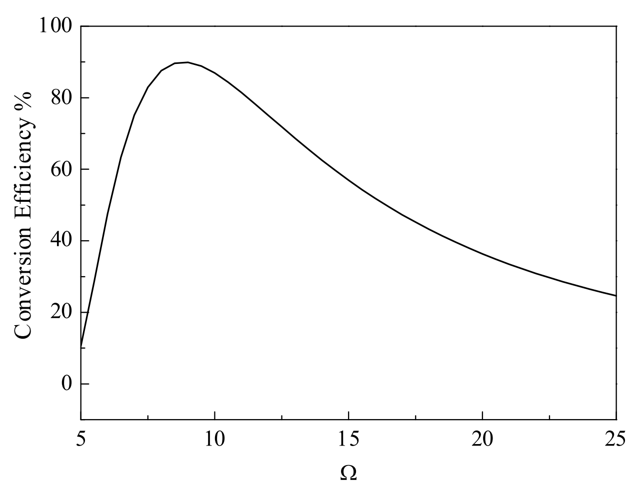

FIG. 2: Quantum efficiency of optical field conversion from $\lambda_1=795nm$ into $\lambda_2=780nm$ in ^{87}Rb as a function of driving field Rabi frequency Ω (given in units of Γ_2) for the medium length $L=100\mu m$. For the rest of parameters see the text.

In Fig.2 we do not show the region of small values of Ω, where the solution Eq.(7) is not valid. It is seen that the QE reaches its maximum $\sim 90\%$ at $\Omega \sim 8\Gamma$, which is very close to the corresponding value of Ω following from Eq.(8) for equal group velocities. To demonstrate the shape-conserving character of frequency conversion in our scheme we present in Fig.3 the results for two different shapes of initial ω_1 pulse. Fig.3a shows the case of Gaussian shaped input pulse having a duration T, while in Fig.3b the latter is taken as two superimposed Gaussian pulses of the same duration T. It is seen that in both cases the generated ω_2 mode reproduces almost identically the initial form of the input ω_1 pulse, which is evidently a direct consequence of linear relationship between quantum fields in Eqs.(5) and (6).

At other values of Ω the incoming ω_1 field is converted into ω_2 mode not completely, but different amount of mode conversion is attainable, thus enabling redistribution of quantum information between the two quantum fields. By adjusting the driving field intensity, a single-photon state entangled over the two optical modes ω_1 and ω_2 with the given intensities can be generated. In particular, such a state with equal intensities of the modes is created for two values of $\Omega \sim 6\Gamma$ and $\Omega \sim 18\Gamma$ (Fig.4), which, however, are different in their physical content. The higher values of Ω correspond to large group velocities of the pulses, while the parametric coupling is reduced. As a result, during propagation in the medium the ω_1 field has time to be converted into ω_2 mode only partially and, owing to $v_2 < v_1$, the ω_2 pulse is slightly delayed with respect to the signal ω_1 pulse that is just observed in Fig.4 (left column). On the contrary, at small values of Ω the pulses travel slower, while the parametric coupling is enhanced. Consequently, first the ω_1 field is almost completely converted into the ω_2 pulse at small distances (nearly 60 μm), and then the latter is partially transformed back into the ω_1 photon at the end ($100\mu m$) of the medium. This time the newly generated ω_1 pulse is clearly behind the ω_2 pulse (Fig.4, right column) with a delay which is longer than in the previous case. By these any input optical mode is split into two optical modes of different frequencies in a controllable way preserving at the same time the total number of photons, which is determined in each mode by the areas of the corresponding peaks. In the absence of losses, the conservation law for photon numbers follows from Eqs.(5) and (6) as

$$\frac{\partial}{\partial z}(n_1(z) + n_2(z)) = 0 \tag{13}$$

with taking into account that $\langle 0 \mid \hat{\mathcal{E}}_i(z, t \to \pm\infty) \mid \psi_{in}\rangle = 0$. At the input of the medium $n_1(0)$=1 and $n_2(0)$=0, so that from Eq.(13) one has $n_1(z) + n_2(z)$=1.

So far we considered the pulses with a duration comparable to the lifetime of atomic 1 and 2 levels. However, the conditions (2-4) do not limit critically the pulse length provided that the spectral bandwidth of quantum fields is still negligible compared to the separation between the upper atomic levels. These conditions can be easily satisfied also for ultrashort pulses, if low atomic densities (lower by several orders of magnitude) are used [18], thus making our scheme able for efficient frequency conversion in this case as well, which is a permanent need in quantum communication tasks.

Thus, by combining the clear properties of parametric interaction between the photons with that the converted

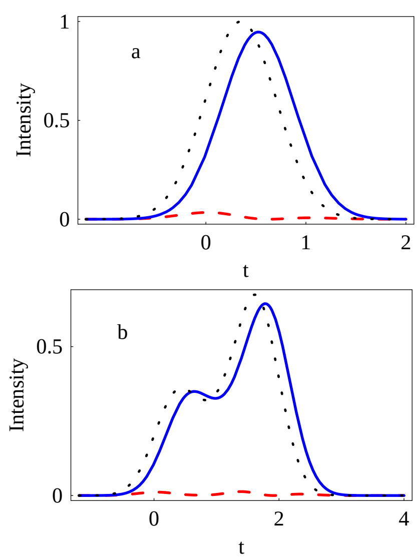

FIG. 3: (Color online) Output shapes of λ_1 (dashed line) and λ_2 (solid line) modes in the cases of Gaussian (a) and double-hump (b) input λ_1 pulses for $\Omega = 8\Gamma$ and other parameters as in Fig.2. The dotted line displays the λ_1 pulse propagating in the medium with $\beta = 0$. The time is given in units of T.

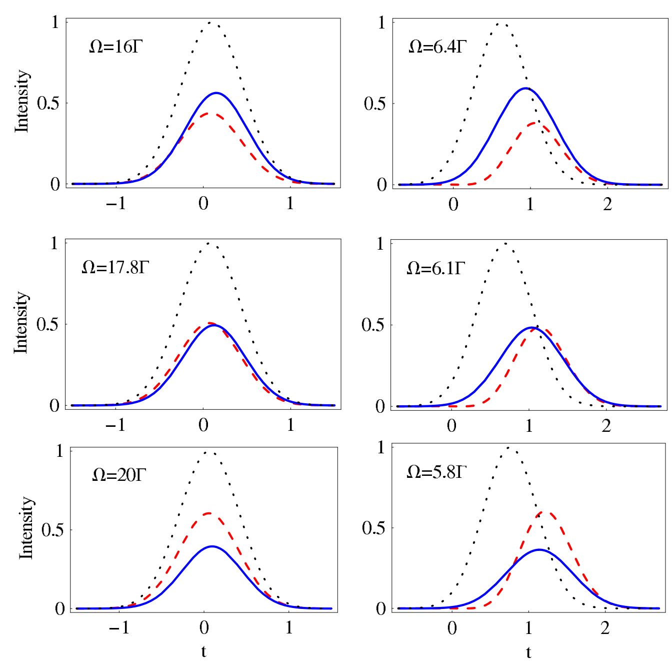

FIG. 4: (Color online) The same as in Fig.3a, but for different values of Ω.

light is within the narrow atomic resonant width, one can significantly expand the ability to distribute the quantum states of narrowband spectra. Above we have demonstrated this possibility in the case of small frequency difference of converted photons that can be used, in particular, for addressable excitation of registers of quantum computers, which may be trapped ions or atoms, atomic ensembles, quantum dots etc. At the same time, there is considerable interest in exploring similar mechanisms that enable coherent conversion of photons with much larger difference in wavelengths. Such a mechanism would enable quantum information to be passed over long distances by telecom fibers, which impose the infrared (IR) wavelengths, and then to be mapped into the states of atoms, which interact resonantly with quantum fields at visible frequencies. Within our approach, the required frequency conversion is easily realized by slightly modifying the interaction configuration in Fig.1 and applying a second classical field on the transition $0 \rightarrow 3$ with the Rabi frequency Ω_0 (Fig.5a). The details of this analysis will be published elsewhere. Here we only outline how the same high conversion efficiency as in the previous case is obtained, but now for ω_1 and ω_2 light modes, which are in the visible and IR regions, respectively. In Fig.5a the quantum fields E_1 and E_2 have equal one-photon detuning Δ, while the driving fields Ω and Ω_0 are exactly resonant with the corresponding transitions. We suppose $\Omega_0 \gg \Gamma_3$, so that the bare atomic states 0 and 3 are split into a doublet of dressed states $\Psi_\pm = (|3\rangle \mp |0\rangle)/\sqrt{2}$, which are well separated by $2\Omega_0$. If now $\Delta = \Omega_0$, then taking into account that in this case the excitation of the atoms from Ψ_- can be neglected, since it is strongly suppressed by the factor of $(\Gamma_3/\Omega_0)^2$, the interaction scheme is reduced to Fig.5b, where Ψ_+ serves as the ground state(correspondingly, for alternative choose $\Delta = -\Omega_0$, the ground state of modified system is Ψ_-), thus making this scheme identical to that of Fig.1. Under the approximations of Eqs.(2,3,4), we obtain for quantum field operators the same Maxwell equations (5,6) with new parametric coupling constant

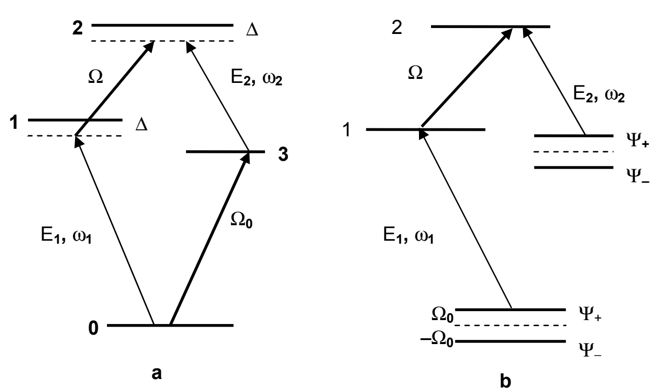

FIG. 5: Modified scheme for conversion of quantum fields E_1 and E_2 with essentially different wavelengths in the basis of bare (a) and dressed (b) atomic states.

$\beta = g_1 g_2 N/(4c\Omega)$, where 4 in denominator emerges from the fact that only a half of the atoms in ground state take part in the process and the atom-field coupling constants g_1 and g_2 decrease by factor $\sqrt{2}$. A key feature of this model is that g_1 and g_2 are approximately equal. Note that this is not the case in Fig.1, if we take here ω_1 and ω_2 in different regions of spectrum, and thereby the conditions (2-4) cannot be satisfied for both quantum fields $E_{1,2}(z,t)$ simultaneously. As an illustration to the modified model in Fig.5, we again consider the sample of ^{87}Rb vapor with the states $5S_{1/2}, 5P_{3/2}, 4D_{3/2}$, and $5P_{1/2}$ being the atomic states 0, 1, 2 and 3 of Fig.5a, respectively. In this case the quantum field wavelengths are $\lambda_1 \sim 780nm$ and $\lambda_2 \sim 1{,}47\mu m$ and $g_2/g_1 \sim 0.96$ that provides high efficiency of IR field conversion into the visible domain and vice versa.

IV. GENERATION OF FREQUENCY ENTANGLED SINGLE-PHOTON STATE

Now we discuss the quantum properties of output single-photon state. We describe it in terms of quantized wave packets at the frequencies ω_1 and ω_2 containing the mean photon numbers $n_i(L) = \frac{c}{L}\int dt \mid \Phi_i(L,t) \mid^2$, where $\Phi_i(L,t) = \langle 0 \mid \hat{\mathcal{E}}_i(L,t) \mid \psi_{in}\rangle$ are the wave functions of the modes. Using Eq.(12) and the commutation relations [10]

$$[\hat{\mathcal{E}}_i(z,t), \hat{\mathcal{E}}_j^+(z,t')] = \frac{L}{c}\delta_{ij}\delta(t-t') \qquad (14)$$

we obtain the output single-photon state as the eigenstate of total photon number operator

$$(\hat{n}_1(L) + \hat{n}_2(L)) \mid \psi_{out}\rangle = \mid \psi_{out}\rangle$$

which yields

$$\mid \psi_{out}\rangle = \frac{c}{L}\int dt[\Phi_1(L,t)\hat{\mathcal{E}}_1^+(L,t) + \Phi_2(L,t)\hat{\mathcal{E}}_2^+(L,t)] \mid 0\rangle \qquad (15)$$

Let us introduce the operators of creation of single-photon wave packets at frequencies ω_i associated with mode functions $\Phi_i(L,t)$, where i labels the members of the denumerably infinite set. For $i = 1,2$ they are given by

$$\hat{c}_{1,2}^+ = N_i^{1/2}\int dt \Phi_{1,2}(L,t)\hat{\mathcal{E}}_{1,2}^+(L,t) \qquad (16)$$

with the normalization constants

$$N_i = \frac{c}{L}\left(\int dt \mid \Phi_1(L,t) \mid^2\right)^{-1}$$

These operators create the single-particle states in the usual way by acting on the vacuum state $\mid 0\rangle$

$$\hat{c}_i^+ \mid 0\rangle = \mid 1_i\rangle \qquad (17)$$

and have the standard boson commutation relations

$$[\hat{c}_i, \hat{c}_j^+] = \delta_{ij} \tag{18}$$

following from Eq.(14).

Note that this definition of quantized wave packets is only useful, if the mode spectra are much narrower compared to the mode spacing that has been suggested from the very beginning. Now, for the algebra (18) we choose the representation of infinite product of all vacua

$$| 0\rangle = \prod_i | 0_i\rangle =| 0_1\rangle | 0_2\rangle \prod_{i\neq 1,2} | 0_i\rangle. \tag{19}$$

However, since in our problem we deal with two frequency modes, while the other modes are not occupied by the photons and, hence, are not taken into account during the measurements, the vacuum may be reduced to $| 0\rangle =| 0_1\rangle | 0_2\rangle$. Then the single-photon state (15) can be written as

$$| \psi_{out}\rangle = r_1\hat{c}_1^+ | 0_1\rangle | 0_2\rangle + r_2 | 0_1\rangle\hat{c}_2^+ | 0_2\rangle =$$

$$= r_1 | 1_1\rangle | 0_2\rangle + r_2 | 0_1\rangle | 1_2\rangle \tag{20}$$

with $r_{1,2} = \sqrt{n_{1,2}(L)}$.

Thus, in general case of $r_{1,2} \neq 0$, the system produces a photonic qubit, i.e., a single-photon state entangled in two distinct frequency modes with the known wave functions $\Phi_{1,2}(L,t)$. The complete conversion of input ω_1 mode depicted in Fig.3 corresponds to pure output state $| \psi_{out}\rangle =| 0_1\rangle\otimes | 1_2\rangle$.

Finally, we demonstrate the persistence of initial quantum coherence and entanglement within the QFC in a simple case of input single-photon state entangled in two well-separated temporal modes or time-bins [19](the concept of the single-photon entanglement is discussed in [10])

$$| \psi_{in}\rangle = (a | 1_1\rangle_t | 0_1\rangle_{t+\tau} + b | 0_1\rangle_t | 1_1\rangle_{t+\tau})\otimes | 0_2\rangle \tag{21}$$

where $| 0_1\rangle_t$ and $| 1_1\rangle_t$ denote Fock states with zero and one ω_1 photon, respectively, at the time t and $| a |^2 + | b |^2 = 1$. Suppose that the single-photon wave packets $| 1_1\rangle_t$ and $| 1_1\rangle_{t+\tau}$ are characterized by temporal profiles $f_0(t)$ and $f_\tau(t)$, respectively, which are not overlapped in time due to large time shift τ. Then, using the Eqs.(7) and (10), the wave function of output ω_2 mode is readily calculated to be

$$\Phi_2(L,t) = \langle 0 | \hat{\mathcal{E}}_2(L,t) | \psi_{in}\rangle = a\Phi_{2,0}(L,t) + b\Phi_{2,\tau}(L,t)$$

where

$$\Phi_{2,0(\tau)}(L,t) = -i\beta \int_0^L dx f_{0(\tau)}(X)\ J_0(\psi)$$

with $X = t - L/v_2 - x\Delta v_{21}/v_1 v_2$. Consequently, from Eq.(16) the creation operator c_2^+ can be represented as a sum of creation operators of the two temporal modes at ω_2 frequency. Following the procedure discussed above the output state in the case of complete conversion ($r_1 = 0$) is eventually found in the form

$$| \psi_{out}\rangle =| 0_1\rangle\otimes(a | 1_2\rangle_t | 0_2\rangle_{t+\tau} + b | 0_2\rangle_t | 1_2\rangle_{t+\tau}) \tag{22}$$

showing that the initial ω_1 qubit is transformed into another at ω_2 frequency with the same complex amplitudes a and b, thus preserving the original amount of entanglement. Two different methods are available for experimental test of coherence transfer during the frequency conversion. In the first case, following the work [2] the single-photon interference for incoming and outgoing photons can be measured and compared to each other, thus revealing an equal phase periodicity in the two interference patterns in agreement with Eqs.(21) and (22). The second method [1, 5] is based on photon-pair interference; the input ω_1 photon is first nonclassically correlated with another photon at ω_3, which does not take part in the frequency conversion process. Then, after complete conversion of the ω_1 photon into ω_2 photon, the entanglement will arise between the ω_2 and ω_3 photons and, hence, the strong two-photon interference between the latter will demonstrate the transfer of input qubit at ω_1 frequency into another at $\omega_2 \neq \omega_1$ frequency. To realize this program for narrowband fields, as is the case here, we propose to employ the Duan, Lukin, Cirac, and Zoller (DLCZ) protocol [20, 21] for generation of nonclassically correlated pair of Stokes ω_1- and anti-Stokes ω_3-photons. Note that the key features of DLCZ protocol have been confirmed in many experiments, including strong quantum correlations between ω_1 and ω_3 fields [22].

V. CONCLUSIONS

Summarizing, we have proposed and analyzed a simple scheme of parametric frequency conversion of optical quantum information in atomic ensembles. We have demonstrated remarkable properties of this scheme such as minimal loss and distortion of the pulse shape and the persistence of initial quantum coherence and entanglement that make it superior against the previous schemes of the QFC in atomic media based on release of stored light state via EIT. Moreover, the narrowband property of single photons and high efficiency of entanglement generation profit the present mechanism to serve as an ideal candidate for frequency conversion and redistribution of optical information in quantum information processing and quantum networking.

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